Twisted partial actions, partial projective representations and cohomology

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Introduction, survey and some new results in collaboration with

M. Khrypchenko

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Exercise: $$\theta_g(X_{g^{-1}} \cap X_h) = X_g \cap X_{gh}.$$

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$$g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{GL}(2,\mathbb{C}), \qquad \theta_g : z \mapsto \frac{az+b}{cz+d}.$$

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G group, S monoid. A map $\theta : G \to S$ satisfying (i), (ii), (iii) is called a (unital) partial hom. If S algebra, say θ is partial repr.

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$$au_g \cdot bu_h = \theta_g(\theta_g^{-1}(a)b)f(g,h)u_{gh},$$

(see $f(g,h)$ below)

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E. A. Scott, A construction which can be used to produce finitely presented infinite simple groups, *J. Algebra* **90** (1984), 294-322.

J. C. Birget, The groups of Richard Thompson and complexity, *Internat. J. Algebra Comput.* **14** (2004), no. 5-6, 569 - 626.

Birget: word problem (low complexity), generalized word problem (undecidable) etc.

Twisted par. actions

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Two unital twist. par. actions of G *on* $\mathcal{A}$

$$\theta = (\{\mathcal{A}_g\}, \{\alpha_g\}, \{f(g,h)\}) \quad \text{and} \quad \theta' = (\{\mathcal{A}_g\}, \{\alpha'_g\}, \{f'(g,h)\})$$

called equivalent if $\exists$ *function*

$$\varepsilon : G \ni g \mapsto \varepsilon_g \in \mathcal{U}(\mathcal{A}_g) \subseteq \mathcal{A}$$

s. that $\forall g, h \in G$, $a \in \mathcal{A}_{g^{-1}}$ *have*

$\theta'_g(a) = \varepsilon_g \theta_g(a) \varepsilon_g^{-1}$, *and* $f'(g,h) = \varepsilon_g \, \theta_g(\varepsilon_h 1_{g^{-1}}) \, f(g,h) \, \varepsilon_{gh}^{-1}$.

Recalling usual gr. cohomology

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$C^n(G, A)$ is abel. grp with pointwise mult-n:

identity : $e_n(x_1, \dots, x_n) = 1_{x_1} 1_{x_1 x_2} \dots 1_{x_1 \dots x_n}$,

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Schur Multiplier: $$M(G) = \{\forall \sigma\}/\sim,$$

where $\sigma \sim \sigma' \iff \sigma'(x, y) = \sigma(x, y)\frac{\varepsilon(x)\varepsilon(y)}{\varepsilon(xy)}$, for some $\varepsilon : G \to K^*$.

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Let K field. **Projective** (usual) **repr.-s:**

homs. $$\Delta : G \to \mathrm{PGL}(n, K) = \mathrm{GL}(n, K)/\mathfrak{Z}.$$

Equivalently, maps $\Gamma : G \to \mathrm{GL}(n, K)$ s. that $\exists \sigma(x, y) \in K^*$,

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Notice: Associativity $\implies \sigma(y, z)\sigma(x, yz) = \sigma(xy, z)\sigma(x, y)$.

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Have: $$M(G) \cong H^2(G, K^*).$$

(allow matrices of inf. size)

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Given $A, B \in \mathrm{Mat}_n(K)$, define $\quad A \lambda B \iff A = \alpha B$, some $\alpha \in K^*$,

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$$pM_X(G) = \bigcup_{A \in \text{ some par.} G\text{-mods}} H^2(G, A).$$

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Par. modules/Lausch modules

Inverse semigr. cohomology in H. Lausch, *J. Algebra*, (1975).

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Definition

A Lausch S-mod. is com. inv. semigr A with pair (λ, α), where λ is hom. $S \to \mathrm{End}\, A$, $s \mapsto \lambda_s$, and α is iso $E(S) \to E(A)$ s. th.

1. $\lambda_e(a) = \alpha(e)a$ *for all* $e \in E(S)$ *and* $a \in A$,
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Lausch free modules $\rightarrow$ unital free partial actions

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Theorem

Given a unital par G-module A $\exists$ $S = S(G)/\sim$ and a free resolution

$$\dots \stackrel{\partial_{n+1}}{\to} R_n \stackrel{\partial_n}{\to} \dots \stackrel{\partial_2}{\to} R_1 \stackrel{\partial_1}{\to} R_0 \stackrel{\epsilon}{\to} \mathbb{Z}_S \to 0,$$

of $\mathbb{Z}_S$ in the category of par. G-modules (where 0 is the zero of an appropr. abel. subcategory), whose cohomology groups with values in A are isomorphic to $H^n(G, A)$.

Thank you!